MTH 2215 Test 3 - Solutions

Spring 2021

Pat Rossi Name ______________________________

Show CLEARLY how you arrive at your answers.

1. Give the quotient and the remainder when:

   (a) 859 is divided by 25.

   $859 = \underbrace{(34)}_{q}\underbrace{(25)}_{b} + \underbrace{9}_{r}$

The quotient $q = 34$ The remainder $r = 9$

   (b) -481 is divided by $29. 481 \div (29)\,(-17) : -493.0$

   $-481 = \underbrace{(-17)}_{q}\underbrace{(29)}_{b} + \underbrace{12}_{r}$

The quotient $q = -17$ The remainder $r = 12$

2. Evaluate the quantity $85 \pmod{21} \equiv$

   Recall: the value of $a \pmod{m}$ is the **remainder** when a is divided by m.

   $85 = (4)\,(21) + \underbrace{1}_{r}$

$85 \pmod{21} \equiv 1$

3. Convert the decimal (base 10) representation of 341 to the equivalent hexadecimal (base 16) expansion

   To convert to base 16, apply the Division Algorithm to the number, using 16 as a divisor. Note the quotient and the remainder. Apply the Division Algorithm to the quotient, using 16 as a divisor. Repeat with each succeeding quotient.

   The digits of the base 16 representation (hexadecimal) going from Left to Right, are the **remainders – in reverse order.**

   $341_{10} =$

$$341 = (21)(16) + 5$$

$$21 = (1)(16) + 5$$

$$1 = (0)(16) + 1$$

   The Hexadecimal representation of the number (going Left to Right) is the string of remainders in reverse order (1, 5, 5 or 155, using hexadecinal digits)

   i.e. $341_{10} = 155_{16}$

4. Determine whether the integers below are congruent modulo the given number:

Recall: $a \equiv b \pmod{m}$ exactly when $(a - b) = km$ for some integer k

Alternatively: $a \equiv b \pmod{m}$ exactly when a and b have the same remainder when divided by m.

(a) $228 \equiv 58 \pmod{5}$

Observe: $228 = (45)(5) + 3$ and $58 = (11)(5) + 3$

(i.e., 228 and 58 both have a remainder of 3 when divided by 5.

Therefore, $228 \equiv 58 \pmod{5}$

Alternatively: $228 - 58 = 170 = (34)(5)$

i.e., $228 - 58 = (k)(5)$ for $k \in \mathbb{Z}$ $(k = 34)$

Therefore, $228 \equiv 58 \pmod{5}$

(b) $90 \equiv 203 \pmod{17}$

Observe: $90 = (5)(17) + 5$ and $203 = (11)(17) + 16$

(i.e., 90 and 203 do NOT have the same remainder when divided by 17.

Therefore, $90 \not\equiv 203 \pmod{17}$

Alternatively: $90 - 203 = -113 = (-7)(17) + 6$

Hence, $90 - 203 \neq k(17)$ $\forall k \in \mathbb{Z}$

Therefore, $90 \not\equiv 203 \pmod{17}$

5. Convert the decimal (base 10) representation of 121 to the equivalent binary (base 2) expansion

   The easiest way to do this may be to convert the base 10 representation to base 16 representation and then replace each hexadecimal digit with its binary 4-digit equivalent.

   To convert to base 16, apply the Division Algorithm to the number, using 16 as a divisor. Note the quotient and the remainder. Apply the Division Algorithm to the quotient, using 16 as a divisor. Repeat with each succeeding quotient.

   The digits of the base 16 representation (hexadecimal) going from Left to Right, are the **remainders – in reverse order.**

   $121_{10} =$

   $$121 = (7)(16) + 9$$

   $$7 = (0)(16) + 7$$

   The Hexadecimal representation of the number (going Left to Right) is the string of remainders in reverse order

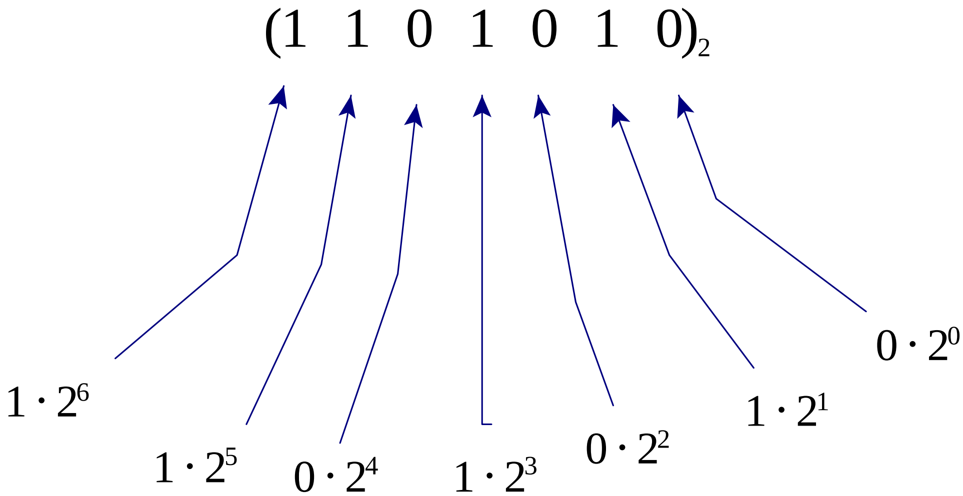
   i.e. $(121)_{10} = (79)_{16}$

   Now replace each hexadecimal digit with it's 4-digit binary equivalent.

   $(121)_{10} = (79)_{16} = (0111 \quad 1001)_2$

   i.e., $(121)_{10} = (0111 \quad 1001)_2$

6. Convert the binary (base 2) representation of $(1101010)_2$ into the equivalent decimal (base 10) expansion



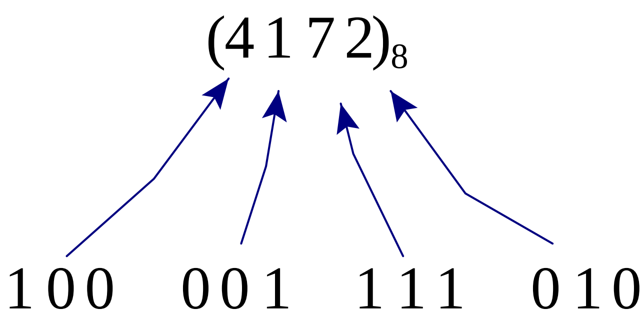
$(1101010)_2 = 1 \cdot 2^6 + 1 \cdot 2^5 + 0 \cdot 2^4 + 1 \cdot 2^3 + 0 \cdot 2^2 + 1 \cdot 2^1 + 0 \cdot 2^0 = 106$

$(1101010)_2 = 106_{10}$

7. Convert the octal (base 8) representation of $(4172)_8$ into the equivalent binary (base 2) expansion

$(4172)_8 =$

Each digit in Base 8 has a unique 3 digit binary representation. Replace each digit in Base 8 with its unique binary equivalent.



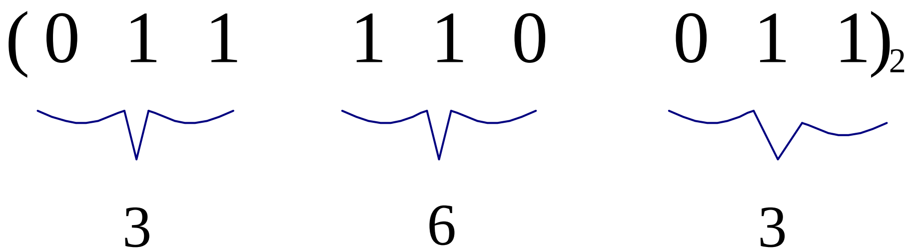
$(4172)_8 = 100\ \ 001\ \ 111\ \ 010_2$

8. Convert the binary (base 2) representation of $(11\ \ 110\ \ 011)_2$ into the equivalent octal (base 8) representation.

   $(11\ \ 110\ \ 011)_2 =$

   The trick here is to group the binary digits into groups of three, adding zeros to the left of the left-most group, as needed.

   Then replace each group of three binary digits with its octal equivalent

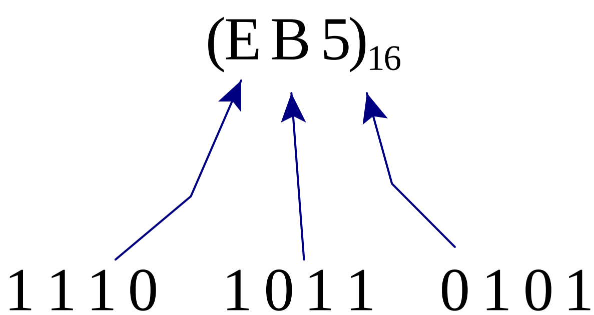


   $(11\ \ 110\ \ 011)_2 = (363)_8$

9. Convert the hexadecimal (base 16) representation of $(EB5)_{16}$ into the equivalent binary (base 2) expansion

   Each digit in Base 16 has a unique 4 digit binary representation. Replace each digit in Base 16 with its unique binary equivalent.

   $(EB5)_{16} =$



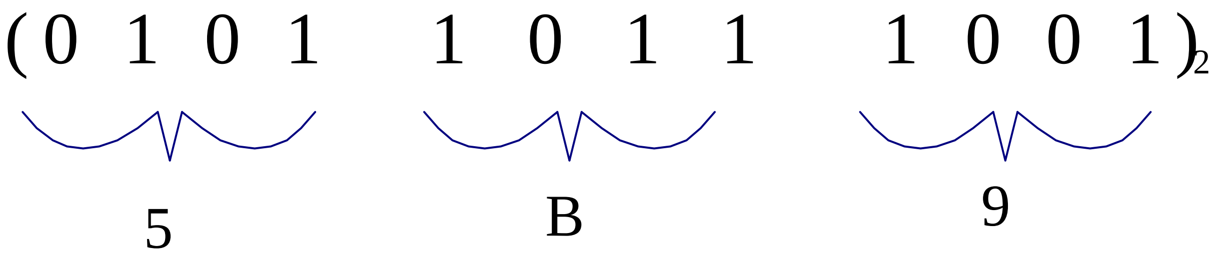
   $(EB5)_{16} = (1110\ \ 1011\ \ 0101)_2$

10. Convert the binary (base 2) representation of $(0101\ \ 1011\ \ 1001)_2$ into the equivalent hexadecimal (base 16) representation

    The trick here is to group the binary digits into groups of four, adding zeros to the left of the left-most group, as needed.

    Then replace each group of four binary digits with its hexadecimal equivalent

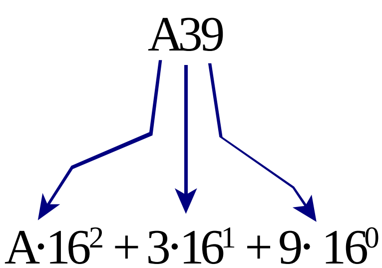
    $(0101\ \ 1011\ \ 1001)_2 =$



$(0101\ \ 1011\ \ 1001)_2 = (5B9)_{16}$

11. Convert the hexadecimal (base 16) representation of $A39_{16}$ into the equivalent base 10 representation

    $A39_{16} =$



$A39_{16} = A \cdot 16^2 + 3 \cdot 16^1 + 9 \cdot 16^0 = 10 \cdot 16^2 + 3 \cdot 16^1 + 9 \cdot 16^0 = 2617_{10}$

i.e., $A39_{16} = 2617_{10}$

12. What is the greatest common divisor of the integers $a = 2^3 \cdot 3^4 \cdot 5^2$ and $b = 2^2 \cdot 3^4 \cdot 5^2$?

To find the $\gcd(a, b)$:

i) Find the prime factors $p_1, p_2, \ldots, p_r$ that both a and b have in common

ii) Find the highest power n_i of each prime factor p_i that appears in both a and b

iii) $\gcd(a, b) = p_1^{n_1} \cdot p_2^{n_2} \cdot \ldots \cdot p_r^{n_r}$

$a = 2^3 \cdot 3^4 \cdot 5^2$

$b = 2^2 \cdot 3^4 \cdot 5^2$

$2, 3, 5$ are prime factors of both a and b

The highest powers of $2, 3$, and 5 that appear in both a and b are $2, 4$,and 2, respectively

$$\gcd(a, b) = 2^2 \cdot 3^4 \cdot 5^2$$