MTH 4441 Homework #2 Groups

Fall 2021

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For Exercises 1-10, decide whether each of the given sets is a group with respect to the given operation. If it is NOT a group, state at least one of the group axioms that fails to hold.

Group Axioms for $(G, *)$

- The Binary Operator $*$ is **closed** on G.
- $*$ is associative
- $(G, *)$ has an identity element
- Each element $x \in G$ has an inverse.

1. The set $\mathbb{Z}^+$ of all positive integers with operation addition.
2. The set $\mathbb{Z}^+$ of all positive integers with operation multiplication.
3. The set $\mathbb{Q}$ of all rational numbers with operation addition.
4. The set $\mathbb{Q}'$ of all irrational numbers with operation addition.
5. The set of all positive irrational numbers with operation multiplication.
6. The set $\mathbb{Q}^+$ of all positive rational numbers with operation multiplication.
7. The set **E** of all even integers with operation addition.
8. The set **E** of all even integers with operation multiplication.
9. The set of all multiples of 5 with operation addition.
10. The set of all multiples of 5 with operation multiplication.

In Exercises 11-12, the given table defines an operation of multiplication on the set $S = \{e, a, b, c\}$. In each case, find a group axiom that fails to hold, and thereby show that S is **not** a group.

11.

$\cdot$	e	a	b	c
e	e	a	b	c
a	a	b	a	b
b	b	c	b	c
c	c	e	c	e

12.

$\cdot$	e	a	b	c
e	e	a	b	c
a	e	a	b	c
b	e	a	b	c
c	e	a	b	c

In exercises, 13-18, let the binary operation be defined on $\mathbb{Z}$ by the rule given. Determine in each case whether $(\mathbb{Z}, *)$ is a group. If it is a group, determine if it is an abelian group. State which conditions, if any fail to hold.

13. $x * y = x + y + 1$

14. $x * y = x + y - 1$

15. $x * y = x + xy$

16. $x * y = xy + y$

17. $x * y = x + xy + y$

18. $x * y = x - y$

In exercises, 19-21, Fill in the group table for $(G, *)$ in as many different ways as possible.

19.

$*$	e	a
e		
a		

20.

$*$	e	a	b
e			
a			
b			

21.

$*$	e	a	b	c
e				
a				
b				
c				