

MTH 4436 Test #1

FALL 2021

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Name _____

1. Define the following: a **divides** b (i.e., $a|b$)
2. Define the following: **Greatest Common Divisor** of a and b (i.e., $\gcd(a, b)$)
3. Define the following: **Relatively Prime**
4. State the Well Ordering Principle
5. State Pascal's Rule
6. State the Binomial Theorem
7. State the Division Algorithm
8. Prove: If $a|b$ and $c|d$, then $ac|bd$.

9. Prove by Induction:

$$1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}$$

$$\text{i.e., } \sum_{i=1}^n i = \frac{n(n+1)}{2}$$

10. Prove:

$$\binom{n}{0} - \binom{n}{1} + \binom{n}{2} - \dots + (-1)^n \binom{n}{n} = 0$$

11. Prove: The sum of any two consecutive triangular numbers is a perfect square.

12. Prove: If $a|b$ and $a|c$, then $a|(bx + cy)$ for arbitrary integers, x and y .

13. Prove that the cube of an integer n is either of the form $9k$, $9k + 1$, or $9k + 8$.

Extra! (10 pts) Prove that the sum of the squares of two odd integers cannot be a perfect square.

Extra! (10 pts) Prove that for any integer n , one of the integers $n, n + 2, n + 4$ is divisible by 3.