

MTH 4436 Test #2

FALL 2021

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Name _____

Instructions. Show CLEARLY how you arrive at your answers

1. Define *prime*

2. Define *Diophantine Equation*

3. State the Fundamental Theorem of Arithmetic

Problems 4-6: **State Three Theorems/Lemmas/Corollaries Regarding Primes.**

4.

5.

6.

7. Prove: There are infinitely many primes.

From Problems 8-9, select one.

8. Prove: The only prime of the form $n^3 - 1$ is 7.

9. If $p \geq 5$ is prime, prove that $p^2 + 2$ is composite. (Hint: Consider the forms that p can take when divided by 6.)

From Problems 10-11, select one.

10. Show that any composite three-digit number must have a prime factor less than or equal to 31.

11. Show that the sum of twin primes p and $p + 2$ is divisible by 12, provided that $p > 3$.

12. Construct the Sieve of Eratosthenes for $n \leq 200$, and list all of the primes less than 200.

	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100
101	102	103	104	105	106	107	108	109	110
111	112	113	114	115	116	117	118	119	120
121	122	123	124	125	126	127	128	129	130
131	132	133	134	135	136	137	138	139	140
141	142	143	144	145	146	147	148	149	150
151	152	153	154	155	156	157	158	159	160
161	162	163	164	165	166	167	168	169	170
171	172	173	174	175	176	177	178	179	180
181	182	183	184	185	186	187	188	189	190
191	192	193	194	195	196	197	198	199	200

13. Test the number 4079 to determine whether it is prime or composite.

14. A Farmer goes to town to buy sheep and goats. Each sheep costs \$54 and each goat costs \$21. If the farmer spends \$3036, determine all of the possible combinations of sheep and goats that he could have purchased.