MTH 2215 Applied Discrete Math - Test #1

SUMMER 2017

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Name: Solutions

Directions: Show CLEARLY how you arrive at your answers!

For Exercises 1-3, Let p and q be as follows:

p: I will take my lunch to school

q: I get up early

1. Translate into symbolic language: "I will get up early and I will take my lunch to school."

$q \wedge p$

2. Translate into symbolic language: "If I get up early, then I will take my lunch to school."

$q \rightarrow p$

3. Translate into symbolic language: "I will take my lunch to school only if I get up early."

$q \rightarrow p$

For Exercises 4-5, Let p and q be as follows:

p: I will celebrate.

q: I get an A.

4. Translate from symbolic language into English: $p \leftrightarrow q$

I will celebrate if and only if I get an A

5. Translate from symbolic language into English: $\neg q \rightarrow \neg p$

If I do not get an A, then I will not celebrate

6. Negate the following statement: "All cows give milk."

$\neg(\forall x \in C, M(x)) \equiv \exists x \in C, (\neg M(x))$

There exists a cow that does not give milk

or Some cows do not give milk

7. Negate the following statement: "Some elephants wear sun-glasses."

$\neg(\exists x \in E, S(x)) \equiv \nexists x \in E, S(x)$

There does not exist an elephant that wears sunglasses

or No elephants wear sunglasses

8. Negate the following statement: "No boats have wheels."

$\neg(\nexists x \in B, W(x)) \equiv \exists x \in B, W(x)$

There exists a boat that has wheels

or Some boats have wheels

9. Negate the following statement: $\exists x \in \mathbb{R},\ \forall y \in \mathbb{R},\ x + y = y$

$$\neg(\exists x \in \mathbb{R},\ \forall y \in \mathbb{R},\ x + y = y) \equiv \forall x \in \mathbb{R} \neg(\forall y \in \mathbb{R},\ x + y = y) \equiv \forall x \in \mathbb{R},\ \exists y \in \mathbb{R},\ \neg(\ x + y = y)$$

$$\equiv \forall x \in \mathbb{R},\ \exists y \in \mathbb{R},\ x + y \neq y$$

Alternatively: $\neg(\exists x \in \mathbb{R},\ \forall y \in \mathbb{R},\ x + y = y) \equiv \nexists x \in \mathbb{R},\ \forall y \in \mathbb{R},\ x + y = y$

$\forall x \in \mathbb{R},\ \exists y \in \mathbb{R},\ x + y \neq y$	OR	$\nexists x \in \mathbb{R},\ \forall y \in \mathbb{R},\ x + y = y$

10. Negate the following statement: $\forall x \in \mathbb{R},\ \exists y \in \mathbb{R},\ x \cdot y = 1$

$$\neg(\forall x \in \mathbb{R},\ \exists y \in \mathbb{R},\ x \cdot y = 1) \equiv \exists x \in \mathbb{R},\ \neg(\exists y \in \mathbb{R},\ x \cdot y = 1) \equiv \exists x \in \mathbb{R},\ \forall y \in \mathbb{R},\ \neg(x \cdot y = 1)$$

$$\equiv \exists x \in \mathbb{R},\ \forall y \in \mathbb{R},\ x \cdot y \neq 1$$

$\exists x \in \mathbb{R},\ \forall y \in \mathbb{R},\ x \cdot y \neq 1$

11. Give the **Converse** and the **Contrapositive** of the statement:

"If I win the election, then I will keep all of my campaign promises."

p → q

Converse: "If I keep all of my campaign promises, then I will have won the election.

q → p

Contrapositive: "If I do not keep all of my campaign promises, then I will not have won the election.

¬q → ¬p

12. Disprove the following statement by providing a counter-example:

For all positive whole numbers n, the number $6n + 1$ is prime.

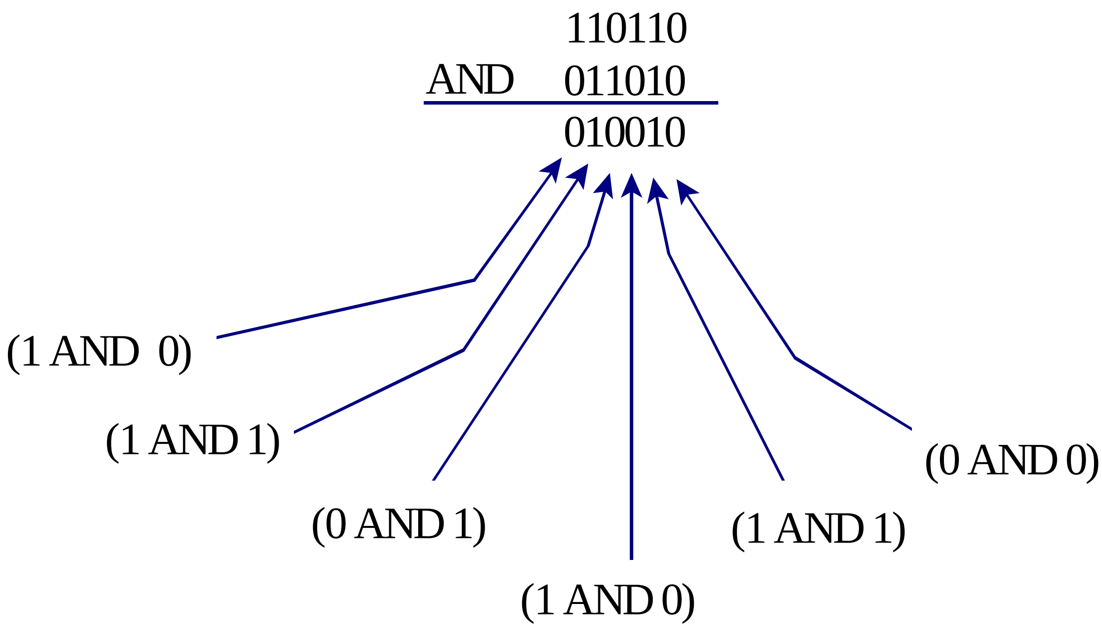
Counterexample: for $n = 4$, $6n + 1 = 6(4) + 1 = 25$, which is not prime.

13. Create the Truth Table for the statement: $((p \wedge q) \vee \neg r) \rightarrow p$

p	q	r	$\neg r$	$p \wedge q$	$(p \wedge q) \vee \neg r$	$((p \wedge q) \vee \neg r) \rightarrow p$
T	T	T	F	T	T	T
T	T	F	T	T	T	T
T	F	T	F	F	F	T
T	F	F	T	F	T	T
F	T	T	F	F	F	T
F	T	F	T	F	T	F
F	F	T	F	F	F	T
F	F	F	T	F	T	F

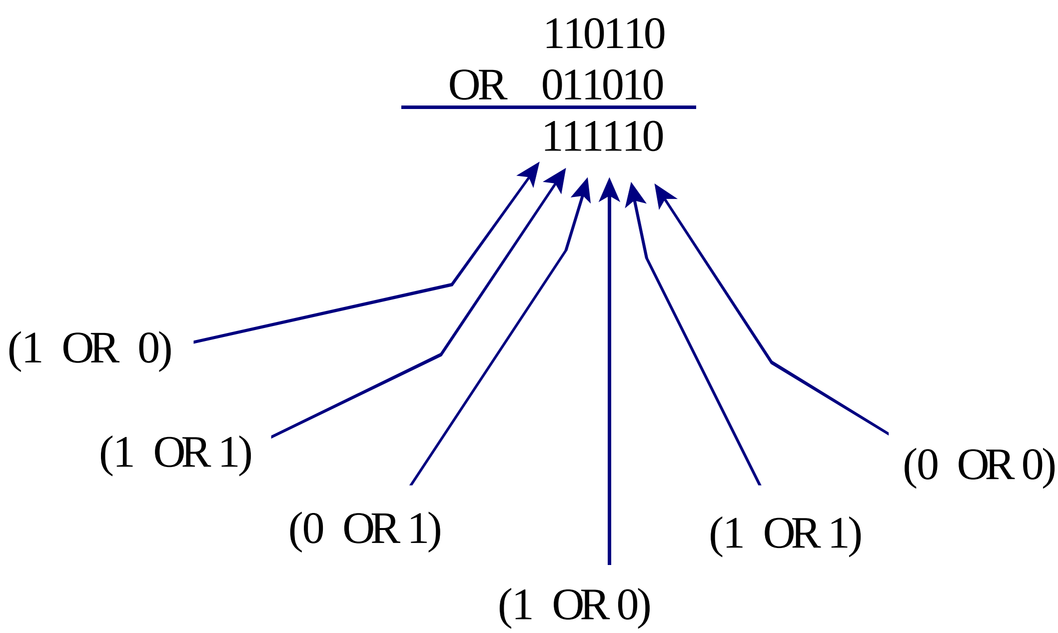
14. Compute the bitwise AND of the two bit strings 110110 and 011010.

110110 AND 011010 ≡ 010010 (The justification is given below)



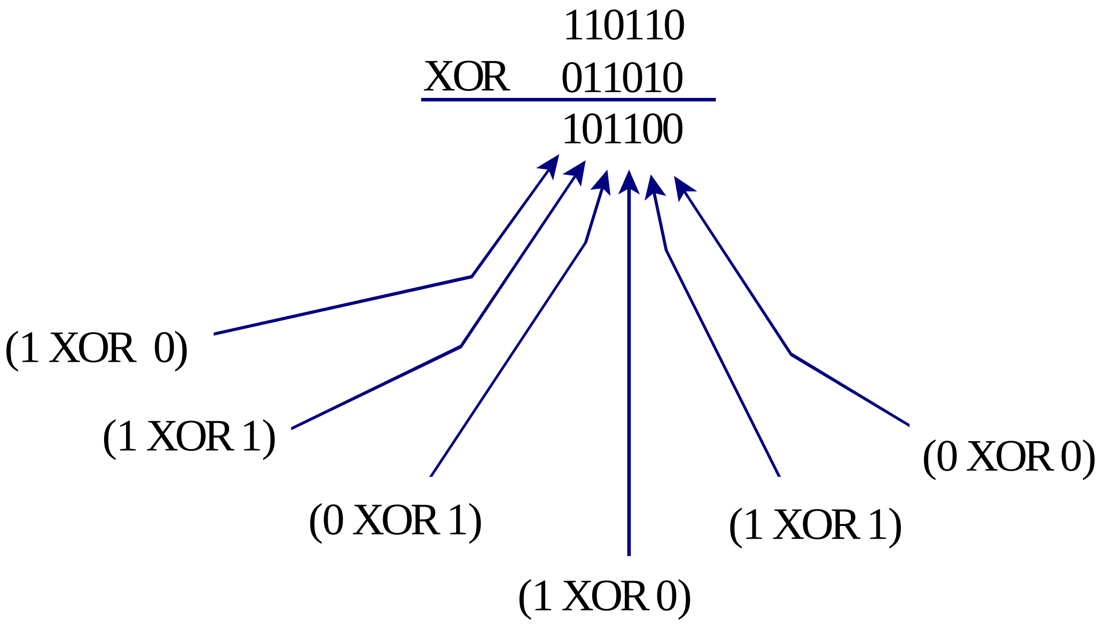
15. Compute the bitwise OR of the two bit strings 110110 and 011010.

110110 OR 011010 ≡ 111110 (The justification is given below)

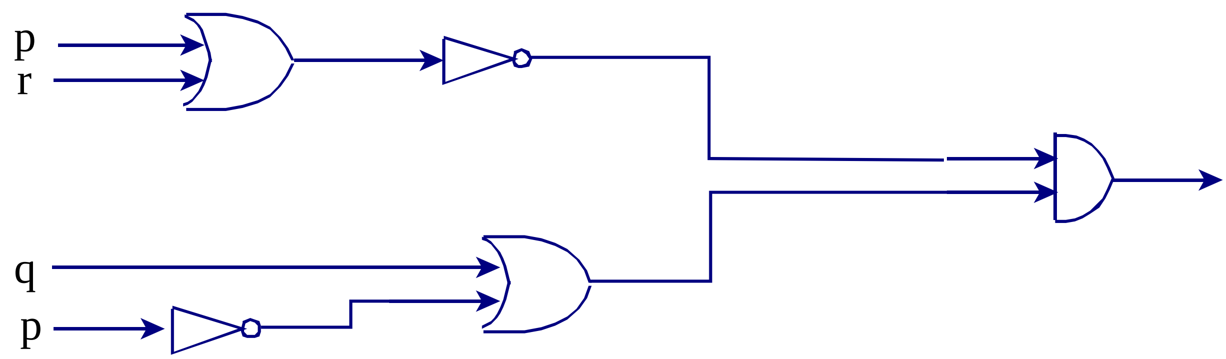


16. Compute the bitwise XOR of the two bit strings 110110 and 011010.

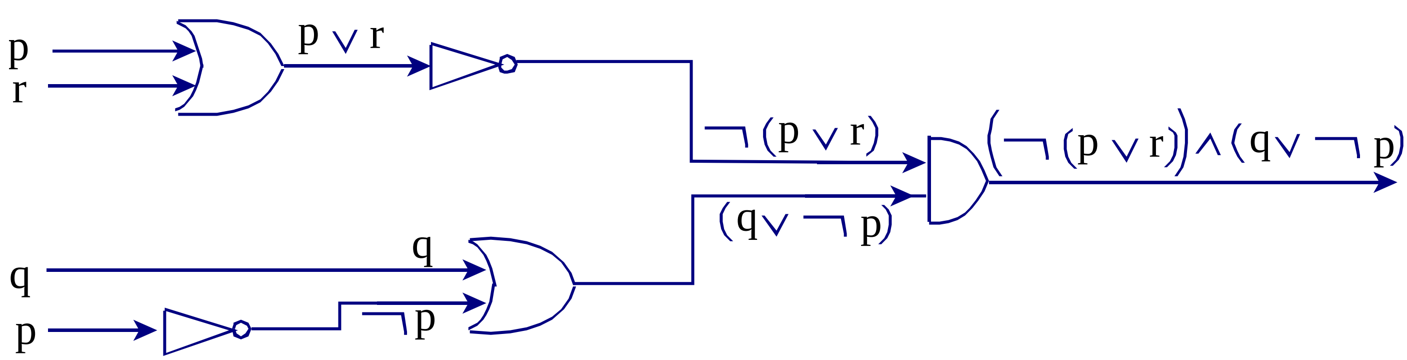
110110 OR 011010 $\equiv$ 101100 (The justification is given below)



17. Compute the output of the combination of Inverter, AND, and OR gates shown below:



We follow the processing of the input step by step, yielding:



18. Determine whether the set of System Specifications is consistent:

s_1: The user paid the subscription fee, but does not enter a valid password.

s_2: Access is granted if the user has paid the subscription fee and has entered a valid password.

s_3: Access is denied if the user has not paid the subscription fee.

s_4: If the user has not entered a valid password, but has paid the subscription fee, then access is not granted.

We represent the simple statements above symbolically, using the following assignments:

p: The user paid the subscription fee

q: The user has entered a valid password

r: Access is granted

Our System specifications are as follows:

s_1: The user paid the subscription fee, but does not enter a valid password. $p \wedge (\neg q)$

s_2: Access is granted if the user has paid the subscription fee and has entered a valid password. $(p \wedge q) \rightarrow r$

s_3: Access is denied if the user has not paid the subscription fee. $(\neg p) \rightarrow (\neg r)$

s_4: If the user has not entered a valid password, but has paid the subscription fee, then access is not granted $(\neg q \wedge p) \rightarrow \neg r$

The System Specifications will be consistent exactly when the conjunction of the specifications is NOT a contradiction. (i.e., exactly when the conjunction of the specifications is True for at least one combination of truth values of p, q, and r.)

p	q	r	$\neg p$	$\neg q$	$\neg r$	s_1: $p \wedge (\neg q)$	$(p \wedge q)$	s_2: $(p \wedge q) \rightarrow r$	s_3: $(\neg p) \rightarrow (\neg r)$	$(\neg q \wedge p)$	s_4: $(\neg q \wedge p) \rightarrow \neg r$	$s_1 \wedge s_2 \wedge s_3 \wedge s_4$
T	T	T	F	F	F	F	T	T	T	F	T	F
T	T	F	F	F	T	F	T	F	T	F	T	F
T	F	T	F	T	F	T	F	T	T	T	F	F
T	F	F	F	T	T	T	F	T	T	T	T	T
F	T	T	T	F	F	F	F	T	F	F	T	F
F	T	F	T	F	T	F	F	T	T	F	T	F
F	F	T	T	T	F	F	F	T	F	F	T	F
F	F	F	T	T	T	F	F	T	T	F	T	F

The fact that "T" appears in the right most column, prevents the conjunction of the system specifications from being a contradiction.

The set of system specifications IS consistent.