

MTH 4441 Test #2

FALL 2021

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Name _____

1. Define - Cyclic group

2. Define - Direct Product of groups $(G, *_G)$ and $(H *_H)$

3. Define - Isomorphism

4. **Prove or Disprove:** $(\mathbb{R}, +)$ is a cyclic group

5. **Prove or Disprove:** $(\mathbb{Q}, +)$ is a cyclic group

6. **Prove:** $(\mathbb{R}, +) \cong (\mathbb{R}^+, \cdot)$

In Exercises 7-10, determine whether the two groups are isomorphic. If they aren't, give at least one reason why. If they are, justify your answer either by exhibiting an isomorphism between the two groups, or by proving that they are isomorphic by some other method.

7. $(2\mathbb{Z}, +)$ and $(3\mathbb{Z}, +)$.

8. $(\mathbb{Z}_2 \times \mathbb{Z}_2, \oplus)$ and the group $(G, *)$ whose group table is given below:

$*$	e	a	b	c
e	e	a	b	c
a	a	b	c	e
b	b	c	e	a
c	c	e	a	b

9. The groups $(G, *)$ and $(H, *)$, whose group tables are given below:

$*$	e	a	b	c
e	e	a	b	c
a	a	b	c	e
b	b	c	e	a
c	c	e	a	b

$*$	e	a	b	c
e	e	a	b	c
a	a	c	e	b
b	b	e	c	a
c	c	b	a	e

10. The groups (\mathbb{Z}_6, \oplus) and $(H, *)$, whose group tables are given below:

\oplus	0	1	2	3	4	5
0	0	1	2	3	4	5
1	1	2	3	4	5	0
2	2	3	4	5	0	1
3	3	4	5	0	1	2
4	4	5	0	1	2	3
5	5	0	1	2	3	4

$*$	e	a	b	c	d	f
e	e	a	b	c	d	f
a	a	b	e	d	f	c
b	b	e	a	f	c	d
c	c	f	d	e	b	a
d	d	c	f	a	e	b
f	f	d	c	b	a	e

11. Given the group table for $(G, *)$, find all of the subgroups of $(G, *)$ and justify your answers. Draw a subgroup diagram for $(G, *)$.

$*$	e	a	b	c	d
e	e	a	b	c	d
a	a	b	c	d	e
b	b	c	d	e	a
c	c	d	e	a	b
d	d	e	a	b	c

12. Construct the group table for (\mathbb{Z}_4, \oplus) , and then find all of the subgroups of (\mathbb{Z}_4, \oplus) and justify your answers. Draw a subgroup diagram for (\mathbb{Z}_4, \oplus) .

13. Calculate the order of the element $(4, 9)$ in the group $\mathbb{Z}_{18} \times \mathbb{Z}_{18}$

14. Calculate the order of the element $(8, 6, 4)$ in the group $\mathbb{Z}_{18} \times \mathbb{Z}_9 \times \mathbb{Z}_8$